International Journal of Theoretical & Applied Sciences, Special Issue-NCRTAST 8(1): 106-108(2016)

ISSN No. (Print): 0975-1718
ISSN No. (Online): 2249-3247

Stability of Fixed Point Iteration Procedures

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(Received 11 April, 2016 Accepted 20 May, 2016)
(Published by Research Trend, Website: www.researchtrend.net)

ABSTRACT: In this paper, we have done the literature review on stability results of fixed point iteration procedure using different contraction conditions in various spaces.

Keywords: stability, iteration procedures, convergence, contraction conditions, fixed point.

I. INTRODUCTION

The fixed point theory is a very important field of research during the last decade. Many research papers are published on this topic during this period making this a very important researching area.

The problem on nonlinear equation are solved by approximating fixed points of a corresponding contractive type mapping. So many methods are there to approximate fixed points. Practically it is important to show that these methods are stable or not. A fixed point iteration is said to be numerically stable if some variations occurs due to approximation at the time of computations, will create variations on the approximate value obtained by this method. The concept of stability is useful in various domains of mathematics which are as follows: Differential Equations, Integral equations, Difference Equations, Numerical Analysis, Game theory etc. Our interest is to review the work done by various authors on stability of fixed point iteration procedure using different contraction conditions.

II. PRELIMINARIES

Types of iteration methods: There exists several methods for approximating fixed points. Let (X, d) be a complete metric space and T: X→X a selfmap of X. Suppose that F_T ={ p Є X, Tp = p} is the set of fixed points of T. There are several iterative processes in the literature for which the fixed points of operators have been approximated over the years by various authors.

1) For x_0 Є X, the sequence $\{x_n\}_{n=1}^{\infty}$ given by

$x_{n+1} = \mathrm{T}x_n$, n ≥ 0 …(1)

Is said the picard iteration

2) Let E be a Banach space and T a mapping from E to E is a self map of E. For x_0 Є E, the sequence $\{x_n\}_{n=1}^{\infty}$ given by $x_{n+1}= (1-\propto_n)\, x_n + \propto_n \mathrm{T}x_n$, …(2) where $\{\alpha_n\}_{n=0}^{\infty}$ is a real sequence in [0,1) such that $\sum_{n=0}^{\infty}\alpha_n=\infty$ is called the Mann iteration. If we put $\alpha_n = 1$ in equation (2) we get the picard iteration.

3) For x_0 Є E, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1-\alpha_n)\, x_n + \alpha_n \mathrm{T}t_n$$
$$t_n = (1-\beta_n)\, x_n + \beta_n \mathrm{T}x_n,$$

where $\{\alpha_n\}$n≥1 and $\{\beta_n\}$n≥1 are sequences in [0, 1) and satisfy $\sum_{n=0}^{\infty}\alpha_n = \infty$ is said the Ishikawa iteration. If we put $\beta_n = 0$ then the ishikawa iteration becomes the mann iteration.

4) For x_0 Є E, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$x_{n+1} = (1-\alpha_n)\, x_n + \alpha_n \mathrm{T}t_n$

$t_n= (1-\beta_n)\, x_n + \beta_n \mathrm{T}z_n,$

$z_n= (1-\gamma_n)\, x_n + \gamma_n \mathrm{T}x_n,$

where $\{\alpha_n\}$n≥1 , $\{\beta_n\}$n≥1, $\{\gamma_n\}$n≥1 are sequences in [0, 1) and satisfy $\sum_{n=0}^{\infty}\alpha_n = \infty$ is said Noor iteration or three step iteration. We know that if we put $\gamma_n = 0$ for each n, then the Noor iteration becomes the Ishikawa iteration.

5) For x_0 Є E, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$x_{n+1} = (1-\alpha_n)\, x_n + \alpha_n \mathrm{T}t_n^1$

$t_n^i= (1-\beta_n^i)\, x_n + \beta_n^i \mathrm{T}t_n^{i+1},\ i = 1,2, \ldots\ldots, k-2$

$t_n^{k-1}= (1-\beta_n^{k-1})\, x_n + \beta_n^{k-1}\mathrm{T}x_n\ , k \geq 2$

Where $\{\alpha_n\}_{n\geq 1}$, $\{\beta_n^i\}$, $i = 1,2, \ldots\ldots, k-1$ are sequences in [0,1) and satisfy

$\sum_{n=0}^{\infty}\alpha_n = \infty$ Is said multistep iteration.

Stability:

The first important result on the stability of a fixed point procedure was studied by Ostrowski [28] in the case of Banach contraction mapping.

Let E be a real Banach space and let T be a mapping defined on E. Let $x_0 \in E$ be arbitrarily chosen and let

$x_{n+1} = f(T, x_n)$ for all n ≥ 0

be an iteration process generating the sequence $\{x_n\}_{n\geq 0}$ in E. Suppose T has at least one fixed point $x^* \in E$ and $x_n \to x^*$ as n→∞. Let $\{z_n\}_{n\geq 0} \subset E$ be any sequence and set

$\epsilon_n = \|z_{n+1} - f(T, z_n)\|$, n≥ 0

The iteration process $\{x_n\}_{n\geq 0}$ is said to be T- stable or stable with respect to T if

$$\log_{n\to\infty} \epsilon_n = 0$$
$$\log_{n\to\infty} \|z_n - x^*\|$$

III. REVIEW

The study of the stability of a fixed point iterative procedure of Banach contraction mappings was first done by Ostrowski [28]. After this study, so many authors developed this subject with certain contractive definitions. Some major developments are as follows-In 1988 Harder and Hicks [10], in 1990,1991 Rhoades , in 1996 Osilike [22] and in 1995 Osilike [23] in 2007 Berinde [6] and in 2002 Berinde [4] ,in 1997 Jachymski [11], in 2006 Olatino, Owojori and Imoru [16], [17] etc. In the year 1995 M.O. Osilike [24] generalised some results of Rhoades [32,33,34] which are already the generalised results of Harder and Hicks [10] and proved the stability results for Ishikawa fixed point iteration procedure. While in the year 1996, M.O.Osilike [25] proved stability of Mann iteration procedure for quasi contractive mappings in Banach spaces. After this, in the year 1997, M. O. Osilike [26] extended this stability of Mann iteration procedure in q-uniformly smooth and p-uniformly convex Banach spaces to Ishikawa iteration method for quasi-contraction maps. While in the year 1997, Chika moore [14] discussed about the T-stability and strong convergence of the Mann and Ishikawa iteration procedures for certain non linear operator equations.Again in the year 1999, M.O. Osilike and A Udomene [27] obtained short proofs of several stability results for fixed point iteration procedures established by Harder and Hicks.

In the year 2002, Zeging Liu *et al* [12] worked on strictly successively hemicontractive mappings and established stable and almost stable iteration procedures in Banach spaces. After this in the year 2003, Zeging Liu *et al* [13] worked on strictly hemi-contractive operators and proved stability in smooth Banach spaces. He also generalised the results given in [8],[29]. In the year 2006, M. O. Olatinwo *et al* [17] worked for the stability of Krasnoselskij and Ishikawa iteration procedure with contractive condition which is the generalised condition used by Berinde [4], Rhoades [33], Harder and Hicks [10] and Osilike [23]. In the year 2008, Olatinwo [19] worked on stability results in normed linear space for two hybrid fixed point iteration of kirk-ishikawa and kirk-mann. He generalised the results of Harder and Hicks [10], Rhoades [32,33], Osilike [23], Berinde[3,4] and the results of various authors [11,16,17,18]. Again in the year 2008, Yuang Qing *et al* [30] worked in metric space to establish T-stability of Picard iteration. While in the year 2009, Memudu Olaposi Olatinwo [20] worked with the contraction conditions which are generalisation of the conditions used by Berinde [4], Imoru and Olatinwo [11] and established the stability results for the picard iteration in complete metric space. In the year 2009, J. O. Olaleru *et al* [15] worked on the stability of different iterative procedures with errors and established almost stability for quasi- contractive maps for Mann,, Ishikawa and Kirk iteration procedures and in the year 2010 Ioana Timis [35] worked on the concept of weak stability for picard iteration procedure satisfying some contraction conditions.

In the year 2011, Memudu Olaposi Olatinwo [21] established the stability results for iteration procedures in convex metric spaces satisfying certain contractive conditions. Also in 2011, Sh. Rezapour *et al* [31] worked on integral type contraction conditions and proved T- stability of Picards iteration procedure and almost T-stability of Mann iteration procedures .While in her paper Zamfirescu maps and it's stability on generalised spaces, Abha Singh [1] Worked on generalised metric spaces and established stability results. In the year 2013, H. Akewe *et al* [2] worked on the concept of strong convergence and stability of multiple iterative procedure. In this paper they also established stability results for Ishikawa, Mann, Noor and Picards iteration procedures. They also generalised the results of Berinde [5], Bosede and Rhoades [7], Imoru and Olatinwo [11] and Osilike [24]. In the year 2014, Faik Gursoy *et al* [9] introduced some new iterative algorithm known as Kirk multistep iteration and established stability results for Kirk multistep and Kirk- SP iterative procedure satisfying certain contraction conditions.

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